Roll Number		

SET C

INDIAN SCHOOL MUSCAT
HALF YEARLY EXAMINATION
MATHEMATICS

CLASS: XII

Sub. Code: 041

Time Allotted: 3 Hrs

22.09.2019

Max. Marks: 80

General Instructions:

(i) All questions are compulsory.
(ii) This question paper contains 36 questions.
(iii) Questions 1- 20 in Section A are MCQ/Very short-answer type questions carrying 1 mark each.
(iv) Questions 21-26 in Section B are short-answer type questions carrying 2 marks each.
(v) Questions 27-32 in Section C are long-answer-I type questions carrying 4 marks each.
(vi) Questions 33-36 in Section D are long-answer-II type questions carrying 6 marks each.

SECTION A

1. Find the area bounded by the curve y = cosx, between x = 0 and x = π. 1

2. Evaluate: $\int log(x^2)\, dx$ 1

3. Find the value of 1

$$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$$

4. If f,g : R→ R be two functions defined as f(x) = $|x| + x$ and g(x) = $|x| - x$, for all x in R, find fog(-5). 1

5. Find the value of $\cos^{-1}\cos\left(\frac{4\pi}{3}\right)$. 1

6 Evaluate: $\int_{-1}^{1} [x]dx$ 1

7. Evaluate: $\int \frac{1+cos2x}{1-cos2x}\, dx$ 1

8. Evaluate : $\int_0^{2\pi} sinx\, dx$ 1

9. Find the area bounded by the lines y = x and x = 2 in the first quadrant. 1

10. A point C in the domain of a function f at which either $f'(c) = 0$ or f is not differentiable is called ---------- . 1

11. The line y = x + 1 is a tangent to the curve $y^2 = 4x$ at the point 1

a) (1, 2) b) (2 , 1) c) (1, -2) d) (-1, 2)

12. f(x) = $\begin{cases} xSin\frac{1}{x}, x \neq 0 \\ k\ ,\ x = 0 \end{cases}$ is continuous at x = 0. Find k. 1

a) 8 b) 1 c) -1 d) 0

13. If $y = x + e^x$, then $\frac{d^2x}{dy^2}$ = ---------- 1

a) $\frac{1}{(1+e^x)^2}$ b) $\frac{-e^x}{(1+e^x)^2}$ c) $\frac{-e^x}{(1+e^x)^3}$ d) e^x

14. Let R be the relation in the set N given by R = $\{(a,b): a = b-2, b > 6\}$.Choose the correct answer. 1

a) (2,4) ∈ R b) (3,8) ∈ R c) (6,8) ∈ R d) (8,7)∈ R

15. f(x) = $\begin{cases} ax^2 + 1\ ,\ x > 1 \\ x + a\ , x \leq 1 \end{cases}$ is differentiable at x = 1, then find the value of a. 1

a) 2 b) 1 c) 0 d) $\frac{1}{2}$

16. The interval in which $y = x^2e^{-x}$ is increasing is 1

a) $(-\infty, \infty)$ b) $(-2, 0)$ c) $(2, \infty)$ d) $(0, 2)$

17. Let f : R→ R be defined as f(x) = x^4.Choose the correct answer. 1

a) F is one- one onto b) f is many-one onto

c) f is one-one but not onto d) f is neither one-one nor onto.

18. Choose the correct principal value branch of the range of $y = \tan^{-1}x$. 1

a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ c) $[0, \pi]$ d) $(0, \pi)$

19. Find the area bounded by f(x) = |x| , between x = -3 and x = 3. 1

a) 0 b) 18 sq.units c) 9 sq.units d) 6 sq.units

20. Find the derivative of $Sin(x)^3$with respect to $Cos(x)^3$. 1

a) $-tan(x^3)$ b) $-cot(x^3)$ c) $cot(x^3)$ d) $tan(x^3)$

SECTION B

21. Find $\int_1^4 f(x)dx$, $if(x) = \begin{cases} 7x\ ;if\ 1 \leq x \leq 3 \\ 8\ ;\ if: 3 \leq x \leq 4 \end{cases}$ 2

OR

.$Evaluate: \int \frac{5^{(7x-5)}}{5^{(2x+10)}}\ dx$

22. Find the value of k ,if the following function is continuous at 1 2

f(x) = $\begin{cases} k(x^2 - 2)\ ,\ x \leq 1 \\ 4x + 1\ ,\ x > 1 \end{cases}$

23. Find $\frac{dy}{dx}$ if, y = $\sin^{-1}(\frac{1-x^2}{1+x^2})$ 0<x<1 2

24. Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{2}{11}\right) = \tan^{-1}\left(\frac{3}{4}\right)$ 2

OR

Evaluate: $sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$

25. The total cost c(x) associated with the production of x units of an item is given by 2

C(x) = $0.007x^3$-$0.003x^2$+15x +4000.Find the marginal cost when 17units are produced.

26. Evaluate: $\int \sqrt{\frac{a+x}{a-x}} - \sqrt{\frac{a-x}{a+x}}\, dx$ 2

SECTION C

27. Find $\frac{dy}{dx}$, $y = (cosx)^{sinx} + (logx)^x$ 4

28. f: N →N be defined by 4

$f(x) = \begin{cases} x+1, if\ x\ is\ odd \\ x-1, if\ x\ is\ even \end{cases}$ for all$x \in N$, show that f is bijective.

29. Find the intervals in which the functions given below are strictly decreasing or strictly increasing:-

$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ 4

OR

Find the equations of the tangent and normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point, where it cuts x-axis.

30. Evaluate: $\int \frac{x+3}{x^2-2x-5}\, dx$ 4

31. If $f(x) = \begin{cases} x^2+3x+a, & x \le 1 \\ bx+2, & x > 1 \end{cases}$, $is\ differentiable.\ Find\ a\ and\ b.$ 4

OR

If $f(x) = \begin{cases} \frac{1-sin^3x}{3cos^2x}, & x < \frac{\pi}{2} \\ a, & x = \frac{\pi}{2} \\ \frac{b(1-sinx)}{(\pi-2x)^2}, & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, find a and b.

32. Simplify : $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$ 4

SECTION D

33. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base. 6

34. Find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ *and* 6

$(x-2)^2 + y^2 = 4$

OR

Using integration find the area of region bounded by the triangle whose vertices are (1,0),(2,2) and (3,1).

35. Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x}\, dx$ 6

36. Let f: **N**→ ***R*** be a function defined as f(x) = $4x^2$ +12x +15. Show that f:**N**→ ***S***, where **S** is the range of f is invertible .Find the inverse of f. 6

OR

Show that the relation R in the set N of Natural numbers given by

R = $\{(a, b): |a - b| \text{ is a multiple of } 3\}$ is an equivalence relation.

End of the Question Paper